Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	8	1	/	0	1 R	Signature	

Paper Reference(s)

6681/01R

Edexcel GCE

Mechanics M5

Advanced/Advanced Subsidiary

Wednesday 18 June 2014 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2} and give your answer to either two significant figures or three significant figures.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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PEARSON

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1. [*In this question*, **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A bead P of mass 0.2 kg is threaded on a smooth straight horizontal wire. The bead is at rest at the point A with position vector $(4\mathbf{i} - \mathbf{j})$ m. A force $(0.2\mathbf{i} + 0.3\mathbf{j})$ N acts on P and moves it to the point B with position vector $(13\mathbf{i} + 5\mathbf{j})$ m.

Find the speed of P at B.

(5)

Leave blank

Question 1 continued

Q1

(Total 5 marks)

Leave blank

2. A uniform equilateral triangular lamina ABC has mass m and sides of length $\sqrt{3}a$. The lamina is free to rotate in a vertical plane about a fixed smooth horizontal axis L, which passes through A and is perpendicular to the lamina. The midpoint of BC is D. The lamina is held with AD making an angle of 60° with the upward vertical through A and released from rest. The moment of inertia of the lamina about the axis L is $\frac{5ma^2}{4}$

Find the speed of D when AD is vertical.

(8)

Leave blank

Question 2 continued

Q2

(Total 8 marks)

Leave blank

3. A uniform rectangular lamina $ABCD$, where $AB = a$ and $BC = 2a$, has mass $2m$. The lamina is free to rotate about its edge AB, which is fixed and vertical. The lamina is at rest when it is struck at C by a particle P of mass m. The particle P is moving horizontally with speed U in a direction which is perpendicular to the lamina. The coefficient of restitution between P and the lamina is 0.5

Find the angular speed of the lamina immediately after the impact.

(8)

Leave blank

Question 3 continued

Q3

(Total 8 marks)

Leave blank

4. A uniform solid sphere has mass M and radius a. Prove, using integration, that the moment of inertia of the sphere about a diameter is $\frac{2Ma^2}{5}$

[*You may assume without proof that the moment of inertia of a uniform circular disc, of mass m and radius r, about an axis through its centre and perpendicular to its plane is* $\frac{1}{2}mr^2$.]

(8)

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Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 8 marks)

Leave blank

5. A particle moves in a plane so that its position vector **r** metres at time t seconds satisfies the differential equation

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} + (\tan t)\,\mathbf{r} = (\cos^2 t)\,\mathbf{i} - (3\cos t)\mathbf{j}, \qquad 0 \leqslant t < \frac{\pi}{2}$$

When $t = 0$, the particle is at the point with position vector $4\mathbf{j}$ m.

Find **r** in terms of t.

(8)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 8 marks)

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6. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a rigid body at the points with position vectors, $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$ respectively, where

$\mathbf{F}_1 = (2\mathbf{i} - \mathbf{j} + \mathbf{k})$ N $\mathbf{F}_2 = (3\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ N $\mathbf{F}_3 = (-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ N

$\mathbf{r}_1 = (\mathbf{i} - \mathbf{k})$ m $\mathbf{r}_2 = (2\mathbf{i} - \mathbf{j} + \mathbf{k})$ m $\mathbf{r}_3 = (\mathbf{i} + \mathbf{j} - \mathbf{k})$ m

The system of the three forces is equivalent to a single force **R** acting at the point with position vector $(3\mathbf{i} - \mathbf{j} + \mathbf{k})$ m, together with a couple of moment **G**.

(a) Find **R**.

(2)

(b) Find **G**.

(9)

Leave blank

Question 6 continued

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Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 11 marks)

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7. A raindrop absorbs water as it falls vertically under gravity through a cloud. In a model of the motion the cloud is assumed to consist of stationary water particles. At time t, the mass of the raindrop is m and the speed of the raindrop is v. At time $t = 0$, the raindrop is at rest. The rate of increase of the mass of the raindrop with respect to time is modelled as being mkv, where k is a positive constant.

(a) Ignoring air resistance, show from first principles, that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = g - kv^2$$

(5)

(b) Find the time taken for the raindrop to reach a speed of $\frac{1}{2}\sqrt{\left(\frac{g}{k}\right)}$

(4)

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total 9 marks)

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8.

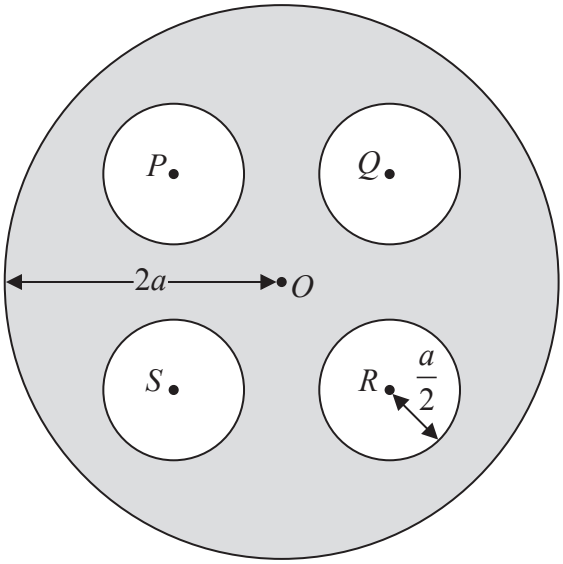

Figure 1

A uniform circular disc of radius $2a$ has centre O. The points P, Q, R and S on the disc are the vertices of a square with centre O and $OP = a$. Four circular holes, each of radius $\frac{a}{2}$, and with centres P, Q, R and S, are drilled in the disc to produce the lamina L, shown shaded in Figure 1. The mass of L is M.

(a) Show that the moment of inertia of L about an axis through O, and perpendicular to the plane of L, is $\frac{55Ma^2}{24}$

(8)

The lamina L is free to rotate in a vertical plane about a fixed smooth horizontal axis which is perpendicular to L and which passes through a point A on the circumference of L. At time t, AO makes an angle θ with the downward vertical through A.

(b) Show that $\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{48g}{151a}\sin\theta$

(4)

(c) Hence find the period of small oscillations of L about its position of stable equilibrium.

(2)

The magnitude of the component, in a direction perpendicular to AO, of the force exerted on L by the axis is X.

(d) Find X in terms of M, g and θ.

(4)

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Question 8 continued

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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

(Total 18 marks)

TOTAL FOR PAPER: 75 MARKS

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